A Synopsis on

"A Study on Optimizing Initial Basic Feasible Solution of Transportation Problem"

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"A Study on Optimizing Initial Basic Feasible Solution of Transportation Problem"

Introduction:

In modern life of human beings the essential factor is the shipping goods from various production units to the customer service points globally.The transportation model deal with purchasing a homogeneous commodity from different sources to numerous customer service points (markets, warehouses etc.). Nowadays the market is highly competitive, various organizations want to deliver the product to the customer in a cost effective way. To meet this challenge, transportation model provides a powerful framework to determine the best ways to deliver goods to the customer[**24**]. The only United States alone has transportation cost for goods approximately 1 trillion dollar /year. Transportation cost becomes an important component for many companies because they spend millions of dollars on transportating goods from supply to demand[**16**].Therefore,it necessitate them to planned their transportation of products from source to service points.The prime objective of this problem is to minimise the transportation cost .It was the first among the linear programming problems which is modelled and studied.The Russian economist Kantorovitch[**2**] studied and developed the dual simplex method in 1930s for finding solution to it. He published a book on it , entitled *"Mathematical Methods in the Organisation and Planning of Production"* in 1939.Hitchcock[**10**] in 1941 in his research article*"The Distribution of a Product from several sources to numerous localities".*Introduced the concept of transportation problem,and developed an algorithm like primal simplex for optimising the solution of transportation problem.

The transportation model is basically a linear programming problem, the solution can be obtained through the simplex method. it was solved by Dantzig in 1951 using simplex method, but due to it's complexity it is not suitable for large scale transportation model.The peculiar structure of transportation model allow us to developed a technique which is computationally more effective than the simplex method.The transportation problem deals with delivering the some goods from sources (e.g.,Factories,Plants etc.) to the destination(e.g.,Warehouse,Markets etc.) with minimum transportation cost, subject to satisfy the demand and supply constraints. Generally sometimes we say that transportation model is a network optimization problem.It is basically based on the assumption that the model is balanced, means that the demand and supply constraints are equal.if the model is unbalanced we can make it balanced by adding dummy source or dummy destination.The transportation model has generally m sources and n destinations with $m+n$ constraint equations,one for each source and each destination[**1**].The transportation problem has $m+n-1$ independent constraint equations out of $m+n$, in other words initial basic feasible solution(IBFS) has $m+n-1$ basic variables and remaining $(m-1)(n-1)$ are non-basic variables .To obtain the optimum solution of transportation problem, one must start from the finding initial basic feasible solution. Because it affects the optimal solution and reduce the number of steps in second step to obtained the optimal solution. Koopman (1949) developed an optimality criterion for basic solution, the early work of Kantorovitch and Koopman received 1975 Nobel prize in economics for their research efforts [**2**].The set of non-negative allocations ($x_{ij} \geq 0$) which satisfies demand and supply constraints, feasible solution is called basic if it contains not more than $m+n-1$ non-negative allocations. Such solution is called non - degenerate basic feasible solution.

The number of non-negative allocations are less than $m+n-1$ then degeneracy occured in the solution, degeneracy can be removed by allocating zero to independent least cost cell in transportation matrix. We have several meta-heuristic approach to find initial basic feasible solution,like Northwest Corner rule (NWCR)[**1-8**],Least Cost Method (LCM)[**1-8**],Vogel's Approximation Method (VAM)[**1-8**] but only VAM gives the optimal solution or closer to optimal solution.The geometry of the initial basic feasible solution $x_{ij} \geq 0$,$(i=1,2,.....,m$,$j=1,2,.....,n)$ exhibit corner point that is not optimal but close to the optimal solution as the emerging convex polytope region[**24**].In transportation problem when the supply and demand at each source and destination is equal to one and the supply is equal to demand an assignment problem is obtained.

Mathematical formulation:

The tabular form of a transportation problem is a cost matrix representing the unit transportation cost c_{ij} , the cost of shipping one unit goods from ith source to jth destination.Let s_i be the quantity of goods available at source i and d_j be the quantity required at destination j.

Destinations / Sources	1	2	.....	n	*supply*
1	$c_{11}(x_{11})$	$c_{12}(x_{12})$	....	$c_{1n}(x_{1n})$	s_1
2	$c_{21}(x_{21})$	$c_{22}(x_{22})$	....	$c_{2n}(x_{2n})$	s_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$c_{m1}(x_{m1})$	$c_{m2}(x_{m2})$	....	$c_{mn}(x_{mn})$	s_m
Demand	d_1	d_2	....	d_n	$\sum_{j=1}^{n} s_i = \sum_{i=1}^{m} d_j$

$$Min.Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = s_i, i = 1,2,3,....,m$$

$$\sum_{i=1}^{m} x_{ij} = d_j, j = 1,2,3,....,n$$

where $x_{ij} \geq 0, \forall i,j$

Notations:

m -Total number of supply

n-Total number of demand

s_i-Supply i

d_j-Demand j

c_{ij}-Transportation cost from the supply i to demand j

x_{ij}-Allocation made from the supply i to demand j

A transportation problem is balanced if the total supply is equal to the total demand.

$$\sum_{j=1}^{n} s_i = \sum_{i=1}^{m} d_j$$

otherwise it is called unbalanced.

Feasible Solution: Any set of non-negative allocations ($x_{ij} \geq 0$) which satisfies the row and column restrictions is called a 'feasible solution '[**4**].

Existence of Feasible Solution: A necessary and sufficient condition to exist the feasible solution to the transportation problem is[**4**]

$$\sum_{j=1}^{n} s_i = \sum_{i=1}^{m} d_j$$

The balanced transportation problem always has a feasible solution[**4**].

Basic Feasible Solution: A feasible solution to a transportation problem is called a "basic feasible solution " if the number of non-negative allocations ($x_{ij} \geq 0$) is equal to $m + n - 1$, where m is the number of rows and n is the number of columns in a transportation problem[**4**].

Thus, a balanced transportation with m sources and n destinations will have at most

$$\binom{mn}{m+n-1} = \frac{(mn)!}{(m+n-1)!(mn-m-n+1)!}$$

basic feasible solution[**6**].

Optimal Solution: A feasible solution (not necessarily basic) that minimize the total cost of transportation[**4**].

Non-degenerate Basic Feasible Solution: A feasible solution to a transportation problem containing m origins and n destinations is said to be"non-degenerate" if it contains $m+n-1$ occupied cells and each allocations is in an independent positions[**4**].

The allocations are in independent positions means it is tricky to form a closed path[**4**].

Closed Path: A path which is in the fashion by allowing transverse and vertical lines occupied only the corner cells[**4**].

Degenerate Basic Feasible Solution: A basic feasible solution in which the number of nonnegative allocations are less than $m + n - 1$[**4**].

Non-negativity A non-negative allocations exist in the problem the values of all variables to be greater than or equal to zero. i.e ($x_{ij} \geq 0$)[**4**].

Review of Literature and Development in the subject:

Several researchers has developed heuristic approach to find an initial basic feasible solution of transportation problem.The well known are Northwest Corner rule (NWCR), Least Cost Method (LCM),and Vogel's Approximation Method (VAM).But we can obtained better initial basic feasible solution by Vogel's Approximation as compared to others.Which is introduced by Reinfeld and Vogel (1958) by defining the penalty as the difference between the least cost and next least cost in each row and column of transportation table.The assignment made to least cost cell corresponding to the highest penalty.Kargul,et al.[**13**], developed a algorithm Avoid Maximum Cost Method (AMCM) to determine the IBFS of transportation model.It is easy to understand and apply,simply the assignment make to the least cost cell of row or column corresponding to the maximum cost cell.It doesn't require any tedious calculations as like the VAM.The main conception behind the method to aside the maximum cost,and make assignment to the least cost to obtained the minimum cost of transportation.Sam'an,et al.[**14**] integrate the modified TDM (Total Difference Method)[**11**] and Kargul-Sahin Approximation Method (KSAM)[**16**] , developed a algorithm to find initial basic feasible solution of transportation problem.In TDM1 calculate penalties only for row but in this method calculate penalties for column also.

Amaliah,et al.[**15**],proposed a new approach to obtain the IBFS of TP called Bilqis Chastine Erma method (BCE), by modifying the JHM(Juman Hoque Method)[**17**]. JHM directly moved the excess unit (EU) from first least cost (FLC) to second least cost (SLC) leads to obtained non optimize solution. BCE adds some procedure to move the EU from FLC to SLC to improve the JHM[**17**].Kargul and Sahin[**16**], proposed a new Kargul-Sahin Approximation Method (KSAM),in this method the Weighted cost matrix by demand (wcd) and Weighted cost matrix by supply (wcs) using proportional demand matrix (pdm) and proportional supply matrix (psm).They will get the two solution, one from wcd and second from wcs. Between the two solutions the minimum will be initial basic feasible solution.Jamali,et al.[**26**],proposed a algorithm called Minimum Demand Method (MDM), first identify the minimum demand in demand row and make assignment to least cost in the corresponding Column.After crossed out the satisfied demand column moved to the next minimize demand in demand row.

Amaliah,et al.[**9**] using the Total Opportunity Cost Matrix (TOCM) and modified TDM1[**11**], developed a new approach Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) to find the initial basic feasible solution of TP.Total Opportunity Cost Matrix in which the entries are sum of the row and column opportunity matrix,it was introduced by Kirca and Satir. The TOCM-MT can be coded in C++ language.TDM1 proposed by Hosseini[**11**] calculate only row penalty is the sum of difference between each cost and least cost.Kaur et al.[**18**] improvise the Maximum Difference Method (MDM) , which is developed by Sood and Jain[**19**].The penalty calculated as the difference between maximum and second maximum cost cell for each row and each column.Morade[**20**] developed Minimum supply and demand method,the allocation made to least cost cell corresponding to minimum supply or demand. if the minimum cost is not unique then arbitrary selected.Azad and Hossain[**21**],using the average of cost cells for row and column developed a algorithm called Average Penalty method.The allocation made to least cost cell of corresponding to maximum Average Row Penalty (ARP) or Average Column Penalty (ACP).Hosseini[**11**],proposed three new effective approach with matlab code to solve large size transportation problems.

The Total Difference Method 1 (TDM1) [**11**] calculate the penalty as the total difference between the least cost and each cost only for the rows.The assignment made to least cost corresponding to highest total difference.The Total Difference Square Method (TDSM)[**11**] calculate penalty as total difference of square of each cost and least cost.The Total Difference Method 2 (TDM2) [**11**] is same as TDM1,the only difference is that TDM2 calculate penalty for row and column.Jude et al.[**22**] used the inverse coefficient relation of mean with standard deviation,and proposed a new approach to find IBFS called Inverse Coefficient of Variation Method (ICVM).The row or column with zero standard deviation can be neglected,the assignment made to least cost cell corresponding to smallest $(CV)^{-1}$.Ahmed et al.[**23**] developed a algorithm called Incessant Allocation Method,it doesn't require a balanced TP. if row and column satisfy simultaneously,find the next smallest cost along the satisfied row/column and assign zero to that cell and crossed out both row and column at a time. Ahmed et al.[**25**] proposed Allocation Table Method (ATM),in which the minimum odd cost (MOC) selected from Transportation Table (TT). if there is no odd cost then divide each cost cell by 2 till obtained at least a odd cost,now this table is Known as allocation table (AT).The selected MOC Subtract from each odd cost of TT,now all the cost values are called as allocation cell value (ACV). Now the allocation made to minimum supply/demand, in the corresponding odd value allocation cell value (ACV).

Rashid[**12**],analyze and developed a simple theorem,if a constant is added or subtracted to each element of the transportation cost matrix,an optimal solution of the original problem remains optimal for the new problem and the total transportation cost is increased or decreased by constant times the total supply or demand.Khan et al.[**27**] use the TOCM (Kirca and Satir) and proposed a new approach called TOCM-SUM.it sum up the row and column entries of TOCM,made allocation to the least cost of corresponding highest sum called pointer cost. Seethalakshmy and Srinivasan[**28**] proposed a method,which calculate the penalty as the difference between maximum and next maximum for each row and difference between maximum and minimum for each column.The assignment made to least cost corresponding to highest penalty.Alkubaisi[**29**] proposed a algorithm in which the median cost is calculate for each row and column,the allocation made to least cost corresponding to maximum median cost. Babu et al.[**30**] modified the Least Cost Method (LCM),and name as Implied Cost Method (ICM).The implied cost determine by multiplying unit transportation cost by minimum of supply or demand,the allocation made to least implied cost.

Soomro et al.[**31**] modify the Vogel's Approximation Method,the penalty calculated as the difference between two maximum cost for each row while the penalty for column as the difference between two minimum cost.The assignment made to smallest shipping cost.Babu et al.[**32**] invent a new approach called Lowest Allocation Method (LAM) ,in which the allocation made to least cost corresponding to minimum supply or demand.Deshmukh[**33**], proposed a method which reduces the all cost values 1 or 0 by subtracting least odd cost from odd cost in matrix or multiplying even cost by 1/2.The allocation made to 0 first corresponding to minimum supply or demand.Paul[**34**],rank the row and column on the basis of lower average value, the assignment made to least cost corresponding to lower average value row or column.

Following are Some of the Major Aims and Objectives of the Research Study:

1. To Study basic concepts of initial basic feasible solution.

2. To Study different algorithms for finding IBFS of transportation problem.

3. A comparative study of IBFS obtained by various methods to optimize the transportation cost.

4. To study recent applications of transportation problem.

5. Try to develop some new techniques to optimize the initial basic feasible solution.

6. Try to refine old techniques to optimize the initial basic feasible solution as compared to current scenario.

7. To study future scope of transportation problem in different fields.

8. Try to thrown some lights on algorithms to test the optimality of IBFS.

9. Try to optimize initial basic feasible solution using softwares like Tora, Lingo and some others.

10. Try to generate matlab code for some algorithms to find IBFS.

Methodology :

The knowledge of concern topic will be obtained through the survey of research papers,various books and review of latest articles from the UGC approved journal list.Also we would like to visit various prominent research institutes in the fields like Dr.BAMU,IIT Bombay,TIFR,Uni.of Mumbai,SGB Uni.Amaravati and IISc etc.to interact with eminent personalities in Mathematics,also would like to attend national,international conferences related to research work.we will try to construct some techniques to optimize initial basic feasible solution at the same time we will study comparitively various optimization techniques.

Importance of Study:

In this research work we focus on methods to optimize in particular minimize the cost of transportation by using initial basic feasible solution.The transportation problem has wide real life application, also in Management Science,Economics,and Engineering and technology.The study can be directly extended practically in the corresponding fields like inventory control, personal assignment,employment scheduling,cash flow,vehicle routing and many others.In transportation model there are two steps for finding an optimal solution,the first one is finding an initial basic feasible solution and second one is testing the optimality.The optimality of IBFS can be checked through the Modified Distribution Method (MODI) and Stepping Stone Method,which is developed by Charnes and Cooper.Most of the times the IBFS is optimal solution of transportation problem, so we called finding an initial basic feasible solution is fundamental steps for obtaining an optimal solution or minimum cost of transportation.

Plan of Research Work :

Sr. No.	Period	Plan of the work	Target to be achieved.
1	First year	Study of the background required for the research topic.	Good command over the subject and clear the course work
2	Second year	Collecting the papers and analyze them.	Try to get some methods for finding IBFS.
3	Third year	Work on some problems to be included in the thesis.	Submit at least two research papers.
4	Fourth year	Publishing some papers and submitting the thesis.	Some more papers and thesis submission.

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